

FIG. 1

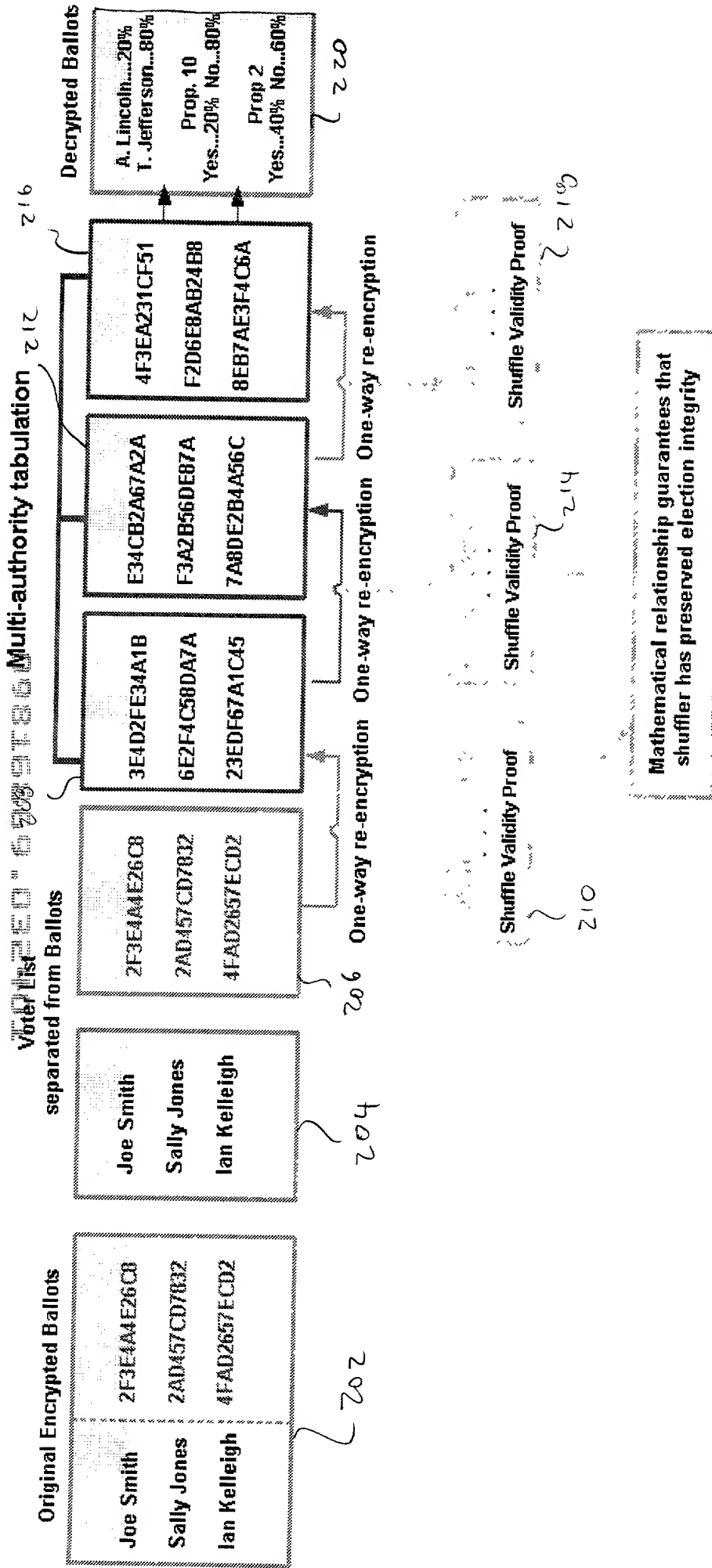


FIG 2

# Scaled Iterated Logarithmic Multiplication Proof

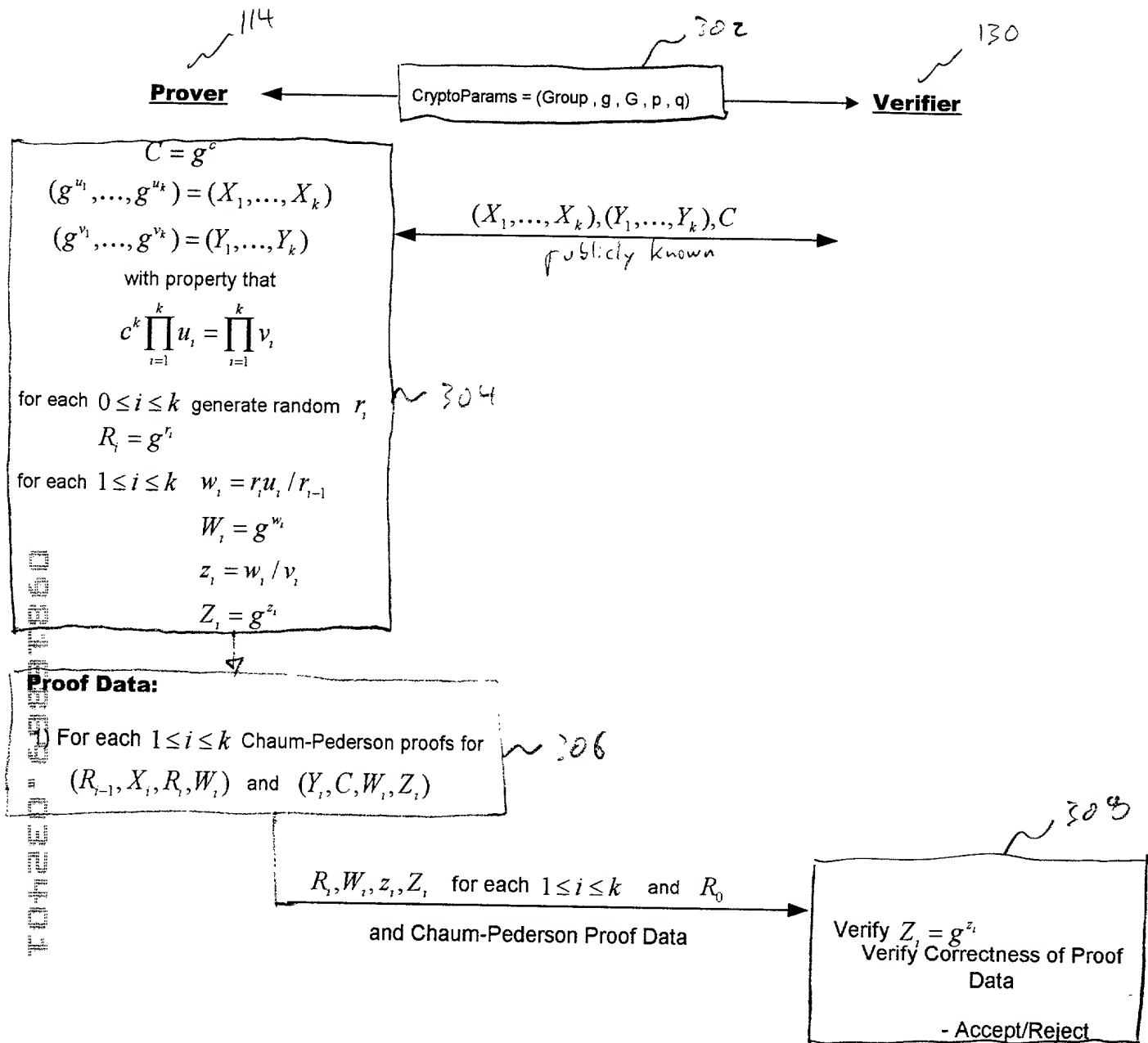


FIG 3

# Simple Shuffle (Shuffler knows exponents)

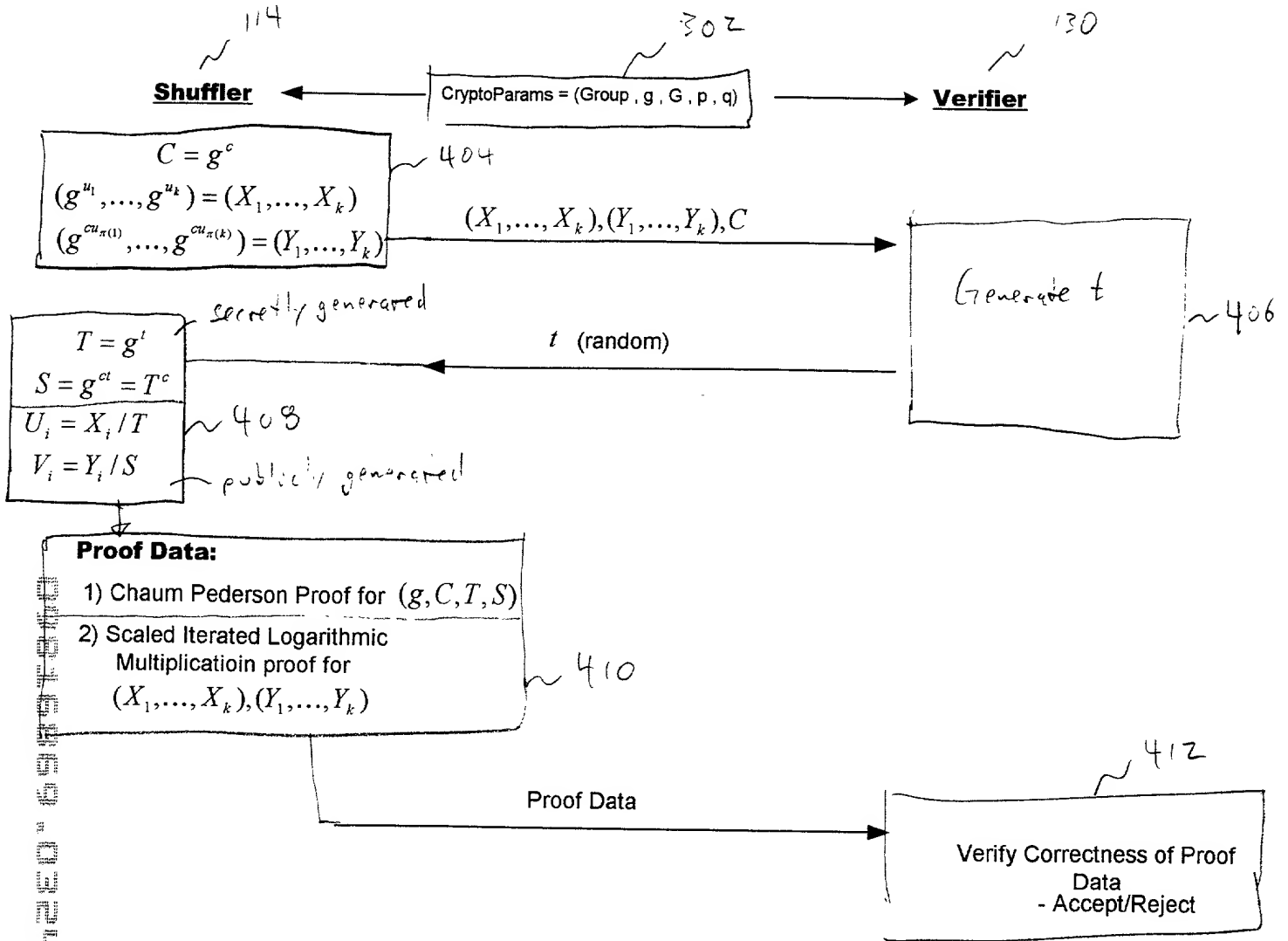


Fig 4

# General Shuffle (Shuffler does not know exponents)

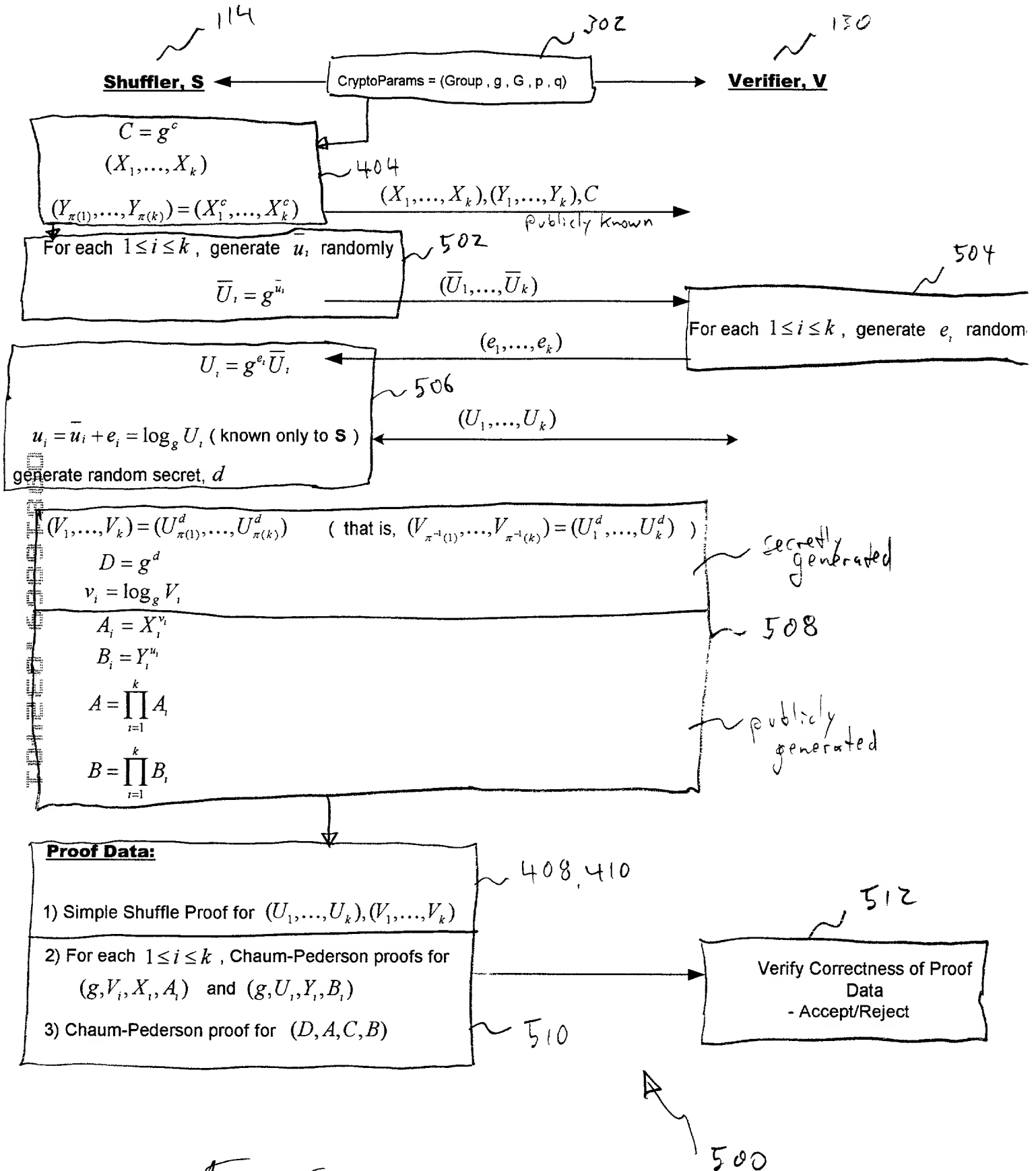


Fig 5

# Anonymous Certificate Distribution (Protocol Variant 1)

## Initialization

CryptoParams = (Group,  $g$ ,  $p$ ,  $q$ ) -- published

$K = H = \text{set of standard public keys} = \{h_1, \dots, h_k\}$

Registrants each know corresponding private key,  $s_j$ , such that  $h_j = g^{s_j}$

$G = g$

## Optional Randomization by Authorities

In sequence, each authority performs verifiable shuffle on  $H$  using  $(G, C = G^e)$  as the shuffle commitment, and returns the shuffled set,  $H'$ , along with the shuffle verification transcript,  $T(H, H', G, C)$ .

If the verification transcript is correct. Registration Server performs the substitutions

$$G = C \quad H = H'$$

and stores the previous values, along with the shuffle verification transcript for audit purposes.

(This can be performed as part of initialization, and/or, at any intermediate stage of anonymous certificate generation.)

## Anonymous Certificate Request and Generation Phase (each registrant in turn)

**Registrant**

Generate Request

anonymous authentication request

**Registration Server**

Retrieve  $G, H$

1) compute shuffle  
(and verification transcript)

2) Generate PKI Certificate Request  
with "random identifying information"

3) Safely store corresponding private key  
for this request.

$T(H, H', G, C)$ ,  $e = cs$  and index,  $1 \leq j \leq k$

$R = \text{PKI Certificate Request}$

1) Check shuffle verification transcript  
2) Check  $h'_j = G^e$

If both checks pass

3) Set  $K = H = H' - \{h'_j\}$

$G = C$

$k = k - 1$

4) Store  $T(H, H', G, C)$   
for audit purposes

5) Digitally sign  $R$  thereby creating  
PKI Certificate,  $\Omega(R)$

Else, if any check fails

$\Omega(R)$

deny request

Loop to beginning of this phase (ready for next anonymous authentication request)

F.14.6

# Anonymous Certificate Distribution (Protocol Variant 2)

## Initialization

CryptoParams = (Group,  $g$ ,  $p$ ,  $q$ ) -- published

$K$  = set of standard public key pairs =  $\{(g_1, h_1), \dots, (g_k, h_k)\}$

$H \subset K$        $J = K - H$

Registrants each know corresponding private key,  $s_j$  such that  $h_j = g_j^{s_j}$

## Optional Randomization by Authorities

In sequence, each authority performs verifiable shuffle (of pairs) on  $K$  using  $(g, C = g^c)$  as the shuffle commitment, and returns the shuffled set,  $K'$ , along with the shuffle verification transcript,  $T(K, K', G, C)$

If the verification transcript is correct. Registration Server performs the substitution

$$K = K'$$

and stores the previous values, along with the shuffle verification transcript for audit purposes.

(This can be performed as part of initialization, and/or, at any intermediate stage of anonymous certificate distribution.)

## Anonymous Certificate Request and Generation Phase (each registrant in turn)

### Registrant

Generate Request

- 1) Select subset  $M \subset H$  of size  $k' \leq k$  and set  $M' = H - M$
- 2) Compute shuffle,  $H'$ , of  $M$  and verification transcript)
- 3) Generate zero knowledge proof,  $P$  that registrant knows exponent  $s$  such that  $(g'_j)^s = h'_j \in H'$  for specified index,  $1 \leq j \leq k'$
- 4) Generate PKI Certificate Request with "random identifying information"
- 5) Safely store private key corresponding to this certificate request

anonymous authentication request

### Registration Server

Retrieve  $H$

$H$  (contains registrant's public key)

$T(M, H', g, C)$ ,  $P$

$R$  = PKI Certificate Request

- 1) Check shuffle verification transcript
- 2) Check  $P$

If both checks pass

- 3) Set  $K = J \cup M' \cup (H' - \{(g'_j, h'_j)\})$   
 $k = k - 1$
- 4) Store  $T(M, H', g, C)$  for audit purposes

- 5) Digitally sign  $R$  thereby creating PKI Certificate,  $\Omega(R)$

$\Omega(R)$

deny request

Else, if any check fails

Loop to beginning of this phase (ready for next anonymous authentication request)

Fig 7